Asymptotically based simulation of the Stokes flow in a layer through periodic flexural plates made of beams

Maxime Krier
Fraunhofer ITWM, Kaiserslautern, Germany
maxime.krier@itwm.fraunhofer.de

Julia Orlik
Fraunhofer ITWM, Kaiserslautern, Germany
julia.orlik@itwm.fraunhofer.de

Grigory Panassenko
Université de Saint-Etienne
grigory.panasenko@univ-st-etienne.fr

Stokes fluid is flowing through a spacer fabric, a porous layer between two parallel hyperplanes with periodically distributed parallel beam latices, which are orthogonal to the hyperplanes. The flow direction is parallel to the hyperplanes and orthogonal to the latices. The top and the bottom of the spacer fabric is insulated for the in- and outflow. The thickness of the spacer fabric is assumed to be one, while the thickness of the latices or porous layers is a small parameter ε. The fluid viscosity is assumed to be $\varepsilon^3 E$, where E is the Young's modulus of the beams. Fluid-solid interaction is considered in the structure. A dimension reduction as $\varepsilon \to 0$ was considered in [1] and the latice-layer is replaced in the paper by its mean surface with a condition: the pressure jump through the surface is proportional to the biharmonic operator in the surface applied to the velocity trace at this surface. The normal component of the limit macroscopic velocity field is an H^2-function of the lattice mean-plane variable and the limit problem is non-local in time. This corresponds to the non-stationarity of the initial problem.

For the numerical computations, a further dimension reduction is performed (see [2], [3], [4]), reducing the elasticity problem to beam equations on one-dimensional lattices and further to a linear algebraic system with 6 unknown degrees of freedom in the nodes of the lattice. Using the equivalence of finite dimensional interpolated norms on segments of the lattice and in the 3D-domain spanned between periodic latices (hexaedral mesh), [5], the nodal solution on the lattices is extended by Q_1 interpolation into the fluid part and the limit problem is solved staying with just 6 degrees of freedom in the lattice nodes. The convergence estimates from the corresponding analysis will be used to estimate the numerical accuracy of the reduced dimension algorithm. Finally, local stresses in the beams and the fluid pressure will be reconstructed as in [2] with the help of interpolated and extended piece-wise polynomial function sequences which are strongly convergent to the solution.

References

[1] J. Orlik, G. Panasenko, R. Stavre, Asymptotic analysis of a viscous fluid layer separated by a thin stiff stratified elastic plate, Applicable Analysis, 100:3, 589-629, (2021)

[2] Griso, G., Khilkova, L., Orlik, J., Sivak, O.: Asymptotic Behavior of Stable Structures Made of Beams. J. Elast (2021). https://doi.org/10.1007/s10659-021-09816-w

[3] Orlik, J., Panasenko, G., Shiryaev, V.: Optimization of textile-like materials via homogenization and dimension reduction. SIAM Multiscale Model. Simul., **14**(2), 637–667 (2016)

[4] Griso, G., Migunova, A., Orlik, J.: Asymptotic Analysis for Domains Separated by a Thin Layer Made of Periodic Vertical Beams. J. Elast., **128**(2), 291–331 (2017)

[5] Falconi, R., Griso, G., Orlik, J.: Anisotropic extension of the unfolding tools, in preparation, 2021