Best Ulam constant of a linear difference equation

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An equation is called Ulam stable if for every approximate solution of it there exists an exact solution near it. We present some results on Ulam stability for some linear difference equations.

In a Banach space X the linear difference equation with constant coefficients $x_{n+p} = a_1 x_{n+p-1} + \ldots + a_p x_n$, is Ulam stable if and only if the roots r_k, $1 \leq k \leq p$, of its characteristic equation do not belong to the unit circle. If $|r_k| > 1$, $1 \leq k \leq p$, we prove that the best Ulam constant of this equation is $\frac{1}{|V|} \sum_{s=1}^{\infty} \left| \frac{V_1}{r_1^s} - \frac{V_2}{r_2^s} + \ldots + \frac{(-1)^{p+1} V_p}{r_p^s} \right|$, where $V = V(r_1, r_2, \ldots, r_p)$ and $V_k = V(r_1, \ldots, r_{k-1}, r_{k+1}, \ldots, r_p)$, $1 \leq k \leq p$, are Vandermonde determinants.