

Tutte's dichromate for signed graphs

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A signed graph is a graph with signed edges (positive or negative). Two signed graphs are considered equivalent if their edge signs differ on a cutset of the graph. Proper colourings and nowhere-zero flows of signed graphs are defined analogously to those of graphs. For graphs, these are both enumerated by evaluations of the Tutte polynomial. For signed graphs, Zaslavsky enumerated proper colourings, and recently DeVos–Rollová–Šámal showed that the number of nowhere-zero flows satisfies a deletion-contraction recurrence, and, independently, Qian–Ren and Goodall–Litjens–Regts–Vena gave a subset expansion formula. We construct a trivariate polynomial invariant of signed graphs that contains both the number of proper colourings and the number of nowhere-zero flows as evaluations: for this three variables are needed, giving a “trivariate Tutte polynomial” for signed graphs. Specializations include Zaslavsky’s bivariate rank-generating polynomial of the (frame matroid of the) signed graph and the Tutte polynomial of the (cycle matroid of the) underlying graph.