Progress on intersecting families

Andrey Kupavskii

Moscow Institute of Physics and Technology, Russia and G-SCOP, CNRS, France

kupavskii@ya.ru

We say that a family of sets is intersecting, if any two of its sets intersect. The Erdos-Ko-Rado theorem characterizes the largest intersecting families of k-element subets of an n-element set, and a lot of studies have been devoted to the following vague question: what structure could a family have, given that it has size close to extremal? This question has many possible answers, depending on the structure we look for, and I am going to discuss several of them during my talk. These will include junta approximations, diversity and covering number.

Such structural results have implications for other related problems, and I will cover at least one such application to the colorings of Kneser graphs. Recall that a Kneser graph is a graph on the collection of all k-element subsets of an n-element set, with two sets connected by an edge when they are disjoint. One of the famous results of Lovász is the topological proof that such graph has chromatic number n - 2k + 2. We discuss the following problem: for which values of n = n(k) we can guarantee that one of the colors is trivial, i.e., it consists of sets containing a fixed element?

Partially based on joint works with Peter Frankl and Sergei Kiselev.