Optimal Hardy weights on the Euclidean lattice

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We investigate the large-distance asymptotics of optimal Hardy weights on $\mathbb{Z}^d$, $d \geq 3$, via the super solution construction. For the free discrete Laplacian, the Hardy weight asymptotic is the familiar $\frac{(d-2)^2}{4}|x|^{-2}$ as $|x| \to \infty$. We prove that the inverse-square behavior of the optimal Hardy weight is robust for general elliptic coefficients on $\mathbb{Z}^d$ in various senses. The proofs leverage Green's function estimates rooted in homogenization theory.