Uniformity for the Number of Rational Points on a Curve

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In 1983, Faltings proved the Mordell Conjecture: a smooth projective curve of genus at least 2 that is defined over a number field K has at most finitely many K-rational points. Several years later Votja gave a new proof. Neither proof provides a procedure to determine the set of rational points, they are ineffective. But the number of rational points can be bounded from above effectively with bounds given by Bombieri, David-Philippon, de Diego, Parshin, Rémond, Vojta, and others. I discuss a result where we show that the number of K-rational points is bounded from above as a function of K, the genus, and the rank of the Mordell-Weil group of the curve's Jacobian. This is joint work with Vesselin Dimitrov and Ziyang Gao and our proof is based on Vojta's approach. Thanks to earlier work by other authors mentioned above, we may reduce to bounding the number of points in a certain height range. For this we develop an inequality for the Néron-Tate height in a family of abelian varieties and use a recent functional transcendence result of Gao.