New results for MaxCut in H-free graphs

Stefan Glock
ETH Zürich
stefan.glock@eth-its.ethz.ch
Oliver Janzer
ETH Zürich
oliver.janzer@math.ethz.ch
Benny Sudakov
ETH Zürich
benny.sudakov@gmail.com

The MaxCut problem asks for the size $\mathrm{mc}(G)$ of a largest cut in a graph G. It is well known that $\mathrm{mc}(G) \geq m/2$ for any m-edge graph G, and the difference $\mathrm{mc}(G) - m/2$ is called the *surplus* of G. The study of the surplus of H-free graphs was initiated by Erdős and Lovász in the 70s, who in particular asked what happens for triangle-free graphs. This was famously resolved by Alon, who showed that in the triangle-free case the surplus is $\Omega(m^{4/5})$, and found constructions matching this bound.

We prove several new results in this area. First, we obtain an optimal bound when H is an odd cycle, adding to the lacunary list of graphs for which such a result is known. Secondly, we extend the result of Alon in the sense that we prove optimal bounds on the surplus of general graphs in terms of the number of triangles they contain. Thirdly, we improve the currently best bounds for K_r-free graphs.

Our proofs combine techniques from semidefinite programming, probabilistic reasoning, as well as combinatorial and spectral arguments.