# Some problems for a mixed type equation fractional order with non-linear loaded term

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Note, that with intensive research on problems of optimal control of the agro-economical system, regulating the label of ground waters and soil moisture, it has become necessary to investigate BVPs for a loaded partial differential equations. Integral boundary conditions have various applications in thermo-elasticity, chemical engineering, population dynamics, etc. In this work we consider parabolic-hyperbolic type equation fractional order involving non-linear loaded term:

$$\begin{aligned} 0 &= u_{xx} - {}_cD_{oy}^{\alpha}u + f_1(x,y;u(x,0)), \quad x>0,\, y>0 \\ 0 &= u_{xx} - u_{yy} + f_2(x,y;u(x+y,0)), \quad x>0,\, y<0, \\ 0 &= u_{xx} - u_{yy} + f_3(x,y;u_y(0,x+y)), \quad x<0,\, y>0 \end{aligned} \tag{1}$$

where $f_i(x,y,u)$, $(i=1,2,3)$ are given functions, ${}_CD_{oy}^{\alpha}u$ Caputo operator fractional order $\alpha$ $(0<\alpha<1)$ (see [1].p.92):

$$\left({}_CD_{ax}^{\alpha}f\right)x = \frac{1}{\Gamma(1-\alpha)}\int\limits_a^x \frac{f'(t)}{(x-t)^{\alpha}}dt, \quad x>a.$$

Let $\Omega \subset R^2$, be domain bounded with segments $B_2A_2$, $A_2A_1$ on the lines $x=l$, $y=h$ at $x>0$, $y>0$; and $A_1C_2$, $C_2B_1$ on the characteristics $x-y=l$, $x+y=0$ of the Eq. (1) at $x>0$, $y<0$, also with the segments $B_1C_1$, $C_1B_2$ on the characteristics $y-x=h$, $x+y=0$ of the Eq. (1) at $x<0$, $y>0$.

We denote as$\Omega_0$ parabolic part of the mixed domain $\Omega$, and hyperbolic parts through $\Omega_1$ at $x>0$ and $\Omega_2$ at $x<0$.

In the domain $\Omega$ $(\Omega=\Omega^+\cup\Omega^-\cup(A_1B_1),)$ we will investigate following

**Problem I.** To find a solution $u(x,y)$ of Eq. (1) from the class of functions: $u(x,y)\in C(\bar{\Omega})\cap C^1\left(\{\overline{\Omega}_2\setminus\overline{A_1C_2}\}\cup\{\overline{\Omega}_1\setminus\overline{C_1B_2}\}\right)\cap C^2(\Omega_2\cup\Omega_1)$; $u_{xx}, {}_CD_{oy}^{\alpha}u\in C\left(\Omega_0\right)$; satisfying boundary conditions:

$$u(l,t)=\varphi_1(y), \;\; 0\le y\le h; \tag{2}$$

$$\frac{d}{dx}u\left(\theta_1(x)\right) = a_1(x)u_y(x,0) + a_2(x)u_x(x,0) + a_3(x)u(x,0) + a_4(x), \ \ 0 \le x < l; \tag{3}$$

$$\frac{d}{dy}u\left(\theta_2(y)\right) = b_1(y)u_x(0,y) + b_2(y)u_y(0,y) + b_3(y)u(0,y) + b_4(y), \ 0 \le y < h; \tag{4}$$

and integral gluing condition:

$$\lim_{y\to+0} y^{1-\alpha}u_y(x,y) = \lambda_1(x)u_y(x,-0) + \lambda_2(x)u_x(x,-0) +$$

$$+\lambda_3(x)\int\limits_0^x r_1(t)u(t,0)dt + \lambda_3(x)u(x,0) + \lambda_4(x), \ 0 < x < l \tag{1}$$

$$u_x(-0,y) = \mu_1(y)u_x(+0,y) + \mu_2(y)u_y(0,y) + \mu_3(y)\int\limits_0^y r_2(t)u(0,t)dt +$$

$$+\mu_4(y)u(0,y) + \mu_5(y), \ 0 < y < h \tag{2}$$

where $\theta_1(x) = \theta_1\left(\frac{x}{2}; -\frac{x}{2}\right)$, $\theta_2(y) = \theta_2\left(-\frac{y}{2}; \frac{y}{2}\right)$, $a_i(x)$, $b_i(y)$, $(i = 1,2,3)$, $\lambda_j(x)$, $\mu_j(y)$ $(j = 1,2,3,4,5)$, $\varphi_1(y)$ are given functions, besides $\sum\limits_{k=1}^{4}\lambda_k^2(x) \neq 0$, $\sum\limits_{k=1}^{4}\mu_k^2(x) \neq 0$, $\sum\limits_{k=1}^{3}a_k^2(x) \neq 0$ and $\sum\limits_{k=1}^{3}b_k^2(x) \neq 0$.

On the certain conditions to given function, we can prove uniqueness of solution of the **Problem I** applying the method of integral energy. Existence of solution, reduced to the Volterra and Fredholm type non linear integral equation respected to $u_y(0,y) = \tau_2'(y)$ and $u(x,0) = \tau_1(x)$ accordingly.

*Keywords:* Loaded equation, Caputo operator, non-local condition, integral gluing condition, non-linear integral equations.