Regular and 'half-regular' maps of negative prime Euler characteristic

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In 2005 A. Breda, R. Nedela and the presenter classified the (fully) regular maps on surfaces with negative prime Euler characteristic; this was the first such classification for an infinite family of surfaces. Extending a 2005 result of M. Belolipetsky and G. Jones, in a 2010 paper by M. Conder, T. Tucker and the presenter a corresponding orientable version of the classification was given for orientably-regular maps of genus $p + 1$ for any prime p.

Algebraically, fully regular maps of valency k and face length ℓ correspond to normal quotients of the full $(2, k, \ell)$-triangle groups presented in the form $\Delta(2, k, \ell) = \langle r_0, r_1, r_2; \ r_0^2, r_1^2, r_2^2, (r_0 r_2)^2, (r_2 r_1)^k, (r_1 r_0)^\ell \rangle$. Orientably-regular maps then arise from normal quotients of the orientation-preserving subgroup $\langle r_0 r_1, r_1 r_2 \rangle$ of index two in $\Delta(2, k, \ell)$. Depending on the parity of k and ℓ, however, the group $\Delta(2, k, \ell)$ may contain up to 7 subgroups of index two, giving rise to further families of 'half-regular' maps in addition to the orientably-regular ones.

The above classification results have generated interest in a similar investigation of the remaining families of 'half-regular' maps. The first family studied from this point of view appears to be the one of *bi-rotary maps* which arise from the index-two subgroup $\langle r_0, r_1 r_2 \rangle$ of $\Delta(2, k, \ell)$ for ℓ even. A classification of bi-rotary maps of negative prime Euler characteristic was completed in 2019 by A. Breda, D. Catalano and the presenter. Recently, another such family of maps, called *edge-biregular* and generated by the subgroup $\langle r_0, r_2, r_1 r_0 r_1, r_1 r_2 r_1 \rangle$ of index two in $\Delta(2, k, \ell)$ for both k, ℓ even, have been investigated in detail by O. Reade (2021). Moreover, in a joint 2021 paper by O. Reade and the presenter we have classified edge-biregular maps on surfaces of negative prime Euler characteristic.

In the talk we will review the previous classification results and present details on the new ones for edge-biregular maps.