On the length of matrix algebras

Olga Markova
Moscow State University
ov_markova@mail.ru

By the length of a finite system of generators for a finite-dimensional algebra over an arbitrary field we mean the least positive integer k such that the products of length not exceeding k span this algebra (as a vector space). The maximum length for the systems of generators of an algebra is referred to as the *length of the algebra*. Apart from intrinsic algebraic importance, the length function has applications, for example, in computing methods of the mechanics of isotropic continua and matrix rational procedures.

The length evaluation can be a difficult problem, since, for example, the length of the full matrix algebra is still unknown (Paz's Problem, 1984). Paz conjectured that the length of any generating set for the algebra of n by n matrices is at most $2n-2$. The question about the length determination was later extended on different matrix sets and subalgebras by Laffey (1986).

First we present a survey of our results on the main properties of length function. In particular, we provide a construction of series of matrix algebras demonstrating that a length of a subalgebra can be larger than the length of the algebra and the difference of their lengths can be arbitrary large. This result partially explains the difficulty of length evaluation.

In this talk we will also show that Paz's conjecture holds under the assumption that the generating set contains a nonderogatory matrix or a matrix with minimal polynomial of degree $n-1$. We will also present linear bounds for the length of generating sets that include a matrix with some restrictions on its Jordan normal form. Having an upper bound, we also provide examples of matrix sets of different type which length achieve the bound $2n-2$.

This talk is based on joint research with Alexander Guterman (Moscow State University), Thomas Laffey and Helena Šmigoc (University College Dublin).