

Control of eigenfunctions on negatively curved surfaces

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Given an L^2 -normalized eigenfunction with eigenvalue λ^2 on a compact Riemannian manifold (M, g) and a nonempty open set subset Ω of M , what lower bound can we prove on the L^2 -mass of the eigenfunction on Ω ? The unique continuation principle gives a bound for any Ω which is exponentially small as λ goes to infinity. On the other hand, microlocal analysis gives a λ -independent lower bound if Ω is large enough, i.e. it satisfies the geometric control condition.

This talk presents a λ -independent lower bound for any set Ω in the case when M is a negatively curved surface, or more generally a surface with Anosov geodesic flow. The proof uses microlocal analysis, the chaotic behavior of the geodesic flow, and a new ingredient from harmonic analysis called the Fractal Uncertainty Principle. Applications include control for Schrodinger equation and exponential decay of damped waves. Joint work with Jean Bourgain, Long Jin, and Stéphane Nonnenmacher.