Unsteady micropolar fluid flow in a thick domain with multiscale oscillating roughness and a subdifferential boundary condition

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Motivated by lubrication problems involving complex fluids we consider an unsteady micropolar fluid flow in a two dimentional thick domain Ω^ε. Following [2] the problem is thus described by a non-linear coupled variational system for the fluid velocity v^ε, the pressure p^ε and the angular micro-rotation field Z^ε. We assume moreover a fluid-solid interface law of friction type modelled by a subdifferential condition (see [3]).

Existence, uniqueness and uniform estimates for $(v^\varepsilon, p^\varepsilon, Z^\varepsilon)$ are derived. Then we assume that the thickness and roughness of Ω^ε are described by multiple separated scales of periodic oscillations i.e.

$$\Omega^\varepsilon = \left\{(z_1, z_2) :\ 0 < z_1 < L,\ 0 < z_2 < \varepsilon^m h^\varepsilon(z_1)\right\}$$

with $h^\varepsilon(z_1) = h(z_1, \frac{z_1}{\varepsilon}, \frac{z_1}{\varepsilon^2}, \cdots, \frac{z_1}{\varepsilon^m})$, $0 < \varepsilon << 1$ and $m \geq 2$.

In order to study the asymptotic behaviour of the flow as ε tends to zero we apply the multiple scale convergence technique for reiterated homogenization problems ([1]). The assumption $m \geq 2$ raises several technical difficulties and leads to a new type of divergence free conditions for the limit velocity which play a crucial role in the derivation of the limit problem.

Finally we prove that the limit velocity and pressure (v^0, p^0) and angular micro-rotation field Z^0 solve a totally decoupled system of elliptic variational inequality on one hand and elliptic partial differential equation on the other hand, where the time variable appears as a parameter. Furthermore v^0, p^0 and Z^0 are uniquely determined through auxiliary well-posed problems.

[joint work with M.Boukrouche and F.Ziane]

References

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