Cyclotomic Association Schemes of Broad Classes and Applications to the Construction of Combinatorial Structures

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In 2010, G. Fernández, R. Kwashira and L. Martínez gave a new cyclotomy on $A = \prod_{i=1}^{n} \mathbb{F}_{q_i}$, where $\mathbb{F}_{q_i}$ is a finite field with q_i elements. They defined a certain subgroup H of the group of units of this product ring A for which the quotient is cyclic. The orbits of the corresponding multiplicative action of the subgroup on the additive group of A are of two types:

- The cyclotomic cosets of the quotient of the group of units of A over the subgroup H.
- The n-tuples with arbitrary non-zero elements in positions indicated by a proper subset S of $\{1, \ldots, n\}$ and zeroes elsewhere.

In this talk, we introduce and study a fusion of a class of asociation schemes derived from the mentioned cyclotomy. The association schemes that we are proposing correspond with a fusion of orbits associated to subsets S of $\{1, \ldots, n\}$ of the same cardinality. We call cyclotomic association schemes of broad classes to these association schemes. The fusion corresponds to the operation of adding the permutations of A induced by the permutations of the symmetric group S_n to the transitive permutation group that determines the original association scheme.

We use these association schemes to obtain sporadic examples and infinite families of difference sets and partial difference sets.