Optimal Drawdowns in Insurance

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In the last years, drawdown measures have attained attention in financial mathematics. Recently, these measures had been applied to actuarial mathematics. Not all of the measures considered make sense in an insurance context. We therefore consider the expected discounted time in the critical area for an infinite time horizon. More specifically, let X_t be the surplus process of an insurance portfolio and denote by $\bar{X}_t = \max\{\bar{x}, \sup_{s \leq t} X_s\}$ the running maximum. The drawdown process is then $D_t = \bar{X}_t - X_t$. The expected discounted time in drawdown is defined as $v^1(\bar{x}-x) = \mathbb{E}[\int_0^\infty e^{-\delta t} \mathbb{I}_{D_t > d} dt]$. Here, δ is a preference parameter meaning drawdowns tomorrow are preferred to drawdowns today. In addition, the insurer can buy proportional reinsurance. We aim to find the optimal reinsurance strategy minimising the expected discounted time in drawdown. We find an explicit solution for a diffusion approximation and also discuss the classical Cramér–Lundberg model.