# Hilbert's 17th problem for noncommutative rational functions

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One of the problems on Hilbert's 1900 list asked whether every positive rational function can be written as a sum of squares of rational functions. Its affirmative resolution by Artin in 1927 was a breakthrough for real algebraic geometry. This talk addresses the analog of this problem for noncommutative rational functions. More generally, a rational Positivstellensatz on matricial sets given by linear matrix inequalities is presented. The crucial intermediate step is an extension theorem on invertible evaluations of linear matrix pencils. A consequence of the Positivstellensatz is an algorithm for eigenvalue optimization of noncommutative rational functions. Lastly, the talk discusses some contrast between polynomial and rational Positivstellensätze.