# Slice-by-slice and global smoothness of slice regular functions

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The concept of slice regular function over the real algebra $\mathbb{H}$ of quaternions is a generalization of the notion of holomorphic function of a complex variable. Let $\Omega$ be an open subset of $\mathbb{H}$, which intersects $\mathbb{R}$ and is invariant under rotations of $\mathbb{H}$ around $\mathbb{R}$. A function $f : \Omega \to \mathbb{H}$ is slice regular if it is of class $\mathcal{C}^1$ and, for all complex planes $\mathbb{C}_I$ spanned by 1 and a quaternionic imaginary unit $I$ ($\mathbb{C}_I$ is a 'complex slice' of $\mathbb{H}$), the restriction $f_I$ of $f$ to $\Omega_I = \Omega \cap \mathbb{C}_I$ satisfies the Cauchy-Riemann equations associated to $I$, i.e., $\overline{\partial}_I f_I = 0$ on $\Omega_I$, where $\overline{\partial}_I = \frac{1}{2}\left(\frac{\partial}{\partial\alpha} + I\frac{\partial}{\partial\beta}\right)$.

We study the continuity and the differential regularity of slice regular functions viewed as solutions of the slice-by-slice differential equations $\overline{\partial}_I f_I = 0$ on $\Omega_I$ and as solutions of their global version $\overline{\vartheta} f = 0$ on $\Omega \setminus \mathbb{R}$.

Our results extend to the slice polyanalytic and monogenic cases.