Virtual Morse-Bott index, moduli spaces of pairs, and applications to topology of smooth four-manifolds

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n Feehan and Leness (2020), we introduced an approach to Morse-Bott theory, called virtual Morse-Bott theory, for Hamiltonian functions of circle actions on closed, real analytic, almost Hermitian spaces. In the case of Hamiltonian functions of circle actions on closed, smooth, almost Kaehler (symplectic) manifolds, virtual Morse-Bott theory coincides with classical Morse-Bott theory due to Bott (1954) and Frankel (1959). Positivity of virtual Morse-Bott indices implies downward gradient flow in the top stratum of smooth points in the analytic space. In this monograph, we apply our method to the moduli space of SO(3) monopoles over a complex, Kaehler surface, we use the Atiyah-Singer Index Theorem to compute virtual Morse-Bott indices of all critical strata (Seiberg-Witten moduli subspaces), and we prove that these indices are positive in a setting motivated by the conjecture that all closed, smooth four-manifolds of Seiberg-Witten simple type obey the Bogomolov-Miyaoka-Yau inequality.