

**Improved bounds for the Kakeya maximal conjecture  
using semialgebraic geometry**

Keith Rogers

*Instituto de Ciencias Matemáticas*

`keith.rogers@icmat.es`

We consider tubes, each pointing in a different direction, and the degree to which they can be compressed by positioning them strategically. The Kakeya problem consists of bounding the measure of the union of the tubes (placed in any position). On the one hand, we will show that the measure of any semialgebraic set that contains the tubes must satisfy the expected lower bound, confirming a conjecture of Guth and Zahl. The proof employs tools from real algebraic geometry including Gromov's algebraic lemma and Tarski's projection theorem. On the other hand, we will use polynomial partitioning to prove that the expected bound holds if there is no algebraic structure at all. Balancing between the two cases yields improved bounds for the Kakeya maximal conjecture in higher dimensions. This is joint work with Hickman, Katz and Zhang.