Flows of nonsmooth vector fields: new results on non uniqueness and commutativity

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Given a vector field in R^d, the classical Cauchy-Lipschitz theorem shows existence and uniqueness of its flow (namely, the solution X(t) of the ODE $X'(t) = b(t, X(t))$ from any initial datum $x \in R^d$) provided the vector field is sufficiently smooth. The theorem looses its validity as soon as v is slightly less regular. However, in 1989, Di Perna and Lions introduced a generalized notion of flow, consisting of a suitable selection among the trajectories of the associated ODE, and they showed existence, uniqueness and stability for this notion of flow for much less regular vector fields.

The talk presents an overview and new results in the context of the celebrated DiPerna-Lions and Ambrosio's theory on flows of Sobolev vector fields, including a negative answer to the following long-standing open question: are the trajectories of the ODE unique for a.e. initial datum in R^d

for vector fields as in Di Perna and Lions theorem? We will exploit the connection between the notion of flow and an associated PDE, the transport equation, and combine ingredients from probability theory, harmonic analysis, and the "convex integration" method for the construction of nonunique solutions to certain PDEs.