Surjective isometries on Banach algebras of Lipschitz maps taking values in a unital C^*-algebra

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Let $\mathcal{A}$ be a unital C^*-algebra. If its center is trivial, i.e. $\mathcal{A} \cap \mathcal{A}' = \mathbb{C}1$, we call it a unital factor C^*-algebra. In this talk, we consider unital surjective complex linear isometries on $\mathrm{Lip}(X, \mathcal{A})$ with $\|\cdot\|_L = \|\cdot\|_\infty + L(\cdot)$. The celebrated Kadison theorem yields that unital surjective linear isometries between unital C^*-algebras are Jordan $*$-isomorphisms. We consider surjective linear isometries on Banach algebras of continuous maps taking values in a C^*-algebra and derive extensions of the Kadison theorem.

In [O. Hatori, K. Kawamura and S. Oi, *Hermitian operators and isometries on injective tensor products of uniform algebras and C^*-algebras,* JMAA, 2019], we proved that if $\mathcal{A}_i$ is a unital factor C^*-algebra for $i = 1, 2$, every surjective linear isometry U from $C(K_1, \mathcal{A}_1)$ onto $C(K_2, \mathcal{A}_2)$ is a weighted composition operator of the form $UF(y) = uV_y(F(\varphi(y)))$, where $\varphi : K_2 \to K_1$ is a homeomorphism, $\{V_y\}_{y \in K_2}$ is a strongly continuous family of Jordan $*$-isomorphisms from $\mathcal{A}_1$ onto $\mathcal{A}_2$, and $u \in C(K_2, \mathcal{A}_2)$ is a unitary element. A main result in this talk is a version of the Banach algebras of all Lipschitz maps of the theorem. Recently in [S. Oi, *Hermitian operators and isometries on algebras of matrix-valued Lipschitz maps,* Linear Multilinear Algebra, 2020], we gave a complete description of surjective linear isometries on $\mathrm{Lip}(X, M_n(\mathbb{C}))$, where $M_n(\mathbb{C})$ is the Banach algebra of complex matrices of order n. Hence the main result is the generalization of it.

In the course of the proof, we characterize hermitian operators on $\mathrm{Lip}(X, E)$ with $\|\cdot\|_L$ for any Banach space E. Note that similar results characterizing hermitian operators on $\mathrm{Lip}(X, E)$ with $\|\cdot\|_M = \max\{\|\cdot\|_\infty, L(\cdot)\}$ have been already obtained in [F. Botelho, J. Jamison, A. Jiménez-Vargas and M. Villegas-Vallecillos, *Hermitian operators on Lipschitz function spaces,* Studia Math., 2013].

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