Tensor Decompositions on Simplicial Complexes: Existence and Applications

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Inspired by the tensor network approach from theoretical quantum physics, we develop a framework to define and analyze invariant decompositions of elements of tensor product spaces. We define an invariant decomposition with indices arranged on a simplicial complex, which is explicitly invariant under a given group action. Several versions of such decompositions also allow to cover positivity of the involved objects. We prove that these decompositions exists for all invariant/positive tensors, after possibly enriching the structure of the simplicial complex. The approach cannot only be applied to tensor products of matrices (as done in quantum physics), but to multivariate polynomials as well. This yields new types of decompositions and complexity measures for polynomials, containing certificates for positivity and symmetries.