About coupled gradient-type quasilinear elliptic systems with supercritical growth

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The aim of this talk is pointing out some recent results on the coupled gradient–type quasilinear elliptic system

$$(P)\quad \begin{cases} -\mathrm{div}(A(x,u)|\nabla u|^{p_1-2}\nabla u) + \frac{1}{p_1}A_u(x,u)|\nabla u|^{p_1} = G_u(x,u,v) & \text{in } \Omega,\\ -\mathrm{div}(B(x,v)|\nabla v|^{p_2-2}\nabla v) + \frac{1}{p_2}B_v(x,v)|\nabla v|^{p_2} = G_v(x,u,v) & \text{in } \Omega,\\ u = v = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is an open bounded domain, p_1, $p_2 > 1$ and $A(x,u)$, $B(x,v)$ are $\mathcal{C}^1$–Carathéodory functions on $\Omega \times \mathbb{R}$ with partial derivatives $A_u(x,u)$, respectively $B_v(x,v)$. Here, $(G_u(x,u,v), G_v(x,u,v)) = \nabla G(x,u,v)$ where $G(x,u,v)$ is a given function on $\Omega \times \mathbb{R}^2$.

Even if the coefficients $A(x,u)$ and $B(x,v)$ make the variational approach more difficult, suitable hypotheses allow us to prove that the weak bounded solutions of problem (P) are critical points of the functional

$$\mathcal{J}(u,v) = \frac{1}{p_1}\int_\Omega A(x,u)|\nabla u|^{p_1}dx + \frac{1}{p_2}\int_\Omega B(x,v)|\nabla v|^{p_2}dx - \int_\Omega G(x,u,v)dx$$

in the Banach space $X = X_1 \times X_2$, where $X_i = W_0^{1,p_i}(\Omega) \cap L^\infty(\Omega)$ for $i = 1, 2$.

Unluckily, classical variational theorems cannot apply to $\mathcal{J}$ in X but, following an approach which exploits the interaction between $\|\cdot\|_X$ and the standard norm on $W_0^{1,p_1}(\Omega) \times W_0^{1,p_2}(\Omega)$, the existence of critical points of $\mathcal{J}$ can be proved by means of a generalized Mountain Pass Theorem.

In particular, if the coefficients $A(x,u)$ and $B(x,v)$ grow in the "right" way then $G(x,u,v)$ can have a suitable supercritical growth and if $\mathcal{J}$ is even then (P) has infinitely many weak bounded solutions.

These results are part of joint works with Caterina Sportelli and Addolorata Salvatore.