A CLT for degenerate diffusions with periodic coefficients, and application to homogenization of linear PDEs

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Let $\mathcal{L}^\varepsilon$, $\varepsilon > 0$, be a second-order elliptic differential operator of the form $\mathcal{L}^\varepsilon = \big(a(\cdot/\varepsilon) + \varepsilon^{-1}b(\cdot/\varepsilon)\big)^{\mathrm{T}}\nabla + 2^{-1}\mathrm{Tr}\big(c(\cdot/\varepsilon)\,\nabla\nabla^{\mathrm{T}}\big)$ with a degenerate (possibly vanishing on a set of positive Lebesgue measure) diffusion coefficient $c(x)$. We first show that the diffusion process associated to $\mathcal{L}^\varepsilon$ satisfies a functional CLT with Brownian limit as $\varepsilon \to 0$, and then by employing probabilistic representation (the Feynman-Kac formula) of the solutions to the elliptic boundary-value and the parabolic initial-value problem we conclude the homogenization result. In the non-degenerate (uniformly elliptic) case these steps can be carried out by combining classical PDE results and the fact that the underlying diffusion process does not show a singular behavior in its motion, that is, it is irreducible. In the case of a degenerate diffusion part, this deficiency is compensated by the assumption that the underlying diffusion process with positive probability reaches the part of the state space where the diffusion term is non-degenerate. Also, in this case it is not clear that we can rely on PDE techniques therefore the proofs are completely based on stochastic analysis tools.