Existence of radial bounded solutions for some quasilinear elliptic equation in $\mathbb{R}^N$

Addolorata Salvatore
Università degli studi di Bari Aldo Moro
addolorata.salvatore@uniba.it

We study the quasilinear equation

$$(P) \quad -\mathrm{div}(A(x,u)|\nabla u|^{p-2}\nabla u)+\frac{1}{p}\, A_t(x,u)|\nabla u|^p+|u|^{p-2}u \;=\; g(x,u) \quad \text{in } \mathbb{R}^N,$$

with $N \geq 3$, $p > 1$, where $A(x,t)$, $A_t(x,t) = \frac{\partial A}{\partial t}(x,t)$ and $g(x,t)$ are Carathéodory functions on $\mathbb{R}^N \times \mathbb{R}$.

Under suitable assumptions on $A(x,t)$ and $g(x,t)$ the problem has a good variational structure, i.e. the weak bounded solutions of problem (P) are critical points of the C^1 functional

$$\mathcal{J}(u) \;=\; \frac{1}{p}\int_{\mathbb{R}^N} A(x,u)|\nabla u|^p dx + \frac{1}{p}\int_{\mathbb{R}^N} |u|^p dx - \int_{\mathbb{R}^N} G(x,u)dx,$$

on the Banach space $X = W^{1,p}(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$, with $G(x,t) = \int_0^t g(x,s)ds$.

In order to overcome the lack of compactness, we assume that the problem has radial symmetry, then we look for critical points of $\mathcal{J}$ restricted to X_r, subspace of the radial functions in X.

Following an approach which exploits the interaction between $\|\cdot\|_X$ and the norm on $W^{1,p}(\mathbb{R}^N)$, we prove the existence of at least one weak bounded radial solution of (P) by applying a generalized version of the Ambrosetti–Rabinowitz Mountain Pass Theorem.

The result is contained in a joint work with Anna Maria Candela.