From branching singularities of minimal surfaces to non-smoothness points on an ice-water interface

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Stefan's problem, dating back to the XIX century, aims to describe the evolution of a block of ice melting in water. Its mathematical analysis experienced few progress until the 1970's, when Duvaut reformulated it as the gradient flow of a nice convex functional. In 1977, Caffarelli proved that the ice-water interface is an smooth surface outside of a certain closed set: the so-called singular set. This was as huge breakthrough. However, methods available back in the 1970's did not allow for a fine description of the structure of the singular set.

During the following 20 years, Almgren developed his theory of branching singularities of minimal surfaces, and in 2008 these methods were applied to the not-so-well-known "thin obstacle problem". This induced, in recent years, a fruitful use of Almgren's methods to study singularities in Stefan's problem. But even with these powerful new tools in hand, we were only halfway to obtaining a fully satisfying description of the non-smoothness points on an ice-water interface...