Embeddings with Eulerian faces II: degree conditions

Mark Ellingham
Vanderbilt University
mark.ellingham@vanderbilt.edu
Joanna Ellis-Monaghan
University of Amsterdam
jellismonaghan@gmail.com

As a natural special case of edge-outer embeddability, we consider the problem of finding maximum genus orientable directed embeddings. We allow some faces to be specified in advance. Digraphs with directed embeddings are necessarily eulerian. If we are given an eulerian digraph and a decomposition of the arcs into edge-disjoint directed walks, then we can regard this as a partial embedding, with the walks as specified face boundaries. If we can complete this to an embedding by adding one more face bounded by an euler circuit, then the embedding will have maximum genus subject to containing the specified faces. We show that this is always possible provided the underlying simple graph of our n-vertex digraph has minimum degree at least $(8n + 1)/9$. This is a broad generalization of results for regular tournaments due to Bonnington et al. (2002) and Griggs, McCourt and Širáň (2020).