

## Geometric constructions for Ramsey-Turán theory

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Combining two classical notions in extremal combinatorics, the study of Ramsey-Turán theory seeks to determine, for integers  $m \leq n$  and  $p \leq q$ , the number  $\text{RT}_p(n, K_q, m)$ , which is the maximum size of an  $n$ -vertex  $K_q$ -free graph in which every set of at least  $m$  vertices contains a  $K_p$ .

Two major open problems in this area from the 80s ask: (1) whether the asymptotic extremal structure for the general case exhibits certain periodic behaviour, resembling that of the special case when  $p = 2$ ; (2) constructing analogues of Bollobás-Erdős graphs with densities other than  $1/2$ .

We refute the first conjecture by witnessing asymptotic extremal structures that are drastically different from the  $p = 2$  case, and address the second problem by constructing Bollobás-Erdős-type graphs using high dimensional complex spheres with *all rational* densities. Some matching upper bounds are also provided.