

Strongly regular graphs satisfying the 4-vertex condition

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A graph Γ satisfies the *t-vertex condition*, when for all triples (T, x_0, y_0) of a *t-vertex graph* T with two distinct distinguished vertices x_0, y_0 , and all pairs of distinct vertices x, y of Γ , where $x \sim y$ if and only if $x_0 \sim y_0$, the number $n(x, y)$ of isomorphic copies of T in Γ , where the isomorphism maps x_0 to x and y_0 to y , does not depend on the choice of the pair x, y . A graph satisfies the 3-vertex condition if and only if it is strongly regular. A graph of order v satisfies the *v-vertex condition* if and only if it is rank 3. There are not many graphs known which satisfy the 4-vertex condition. We discuss several new families of such graphs related to polar spaces. One of our constructions is prolific and shows that the number of graphs satisfying the 4-vertex condition grows hyperexponentially in the number of vertices.