Factorizations of infinite graphs

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Let $\mathcal{F} = \{F_\alpha : \alpha \in \mathcal{A}\}$ be a family of infinite graphs. The Factorization Problem $FP(\mathcal{F}, \Lambda)$ asks whether $\mathcal{F}$ can be realized as a factorization of a given infinite graph Λ, namely, whether there is a factorization $\mathcal{G} = \{\Gamma_\alpha : \alpha \in \mathcal{A}\}$ of Λ such that each Γ_α is a copy of F_α.

Inspired by the results on regular 1-factorizations of infinite complete graphs [1] and on the resolvability of infinite designs [4], we study this problem when Λ is either the Rado graph R or the complete graph $K_\aleph$ of infinite order $\aleph$. When $\mathcal{F}$ is a countable family, we show that $FP(\mathcal{F}, R)$ is solvable if and only if each graph in $\mathcal{F}$ has no finite dominating set. Generalizing the existence result of [2], we also prove that $FP(\mathcal{F}, K_\aleph)$ admits a solution whenever the cardinality $\mathcal{F}$ coincides with the order and the domination numbers of its graphs.

Finally, in the case of countable complete graphs, we show some nonexistence results when the domination numbers of the graphs in $\mathcal{F}$ are finite.

References

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