On the Oberwolfach Problem for single-flip 2-factors via graceful labelings

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The Oberwolfach Problem, posed by Ringel in 1967 and still open, asks for each odd integer $v > 1$ and each 2-regular graph F of order v to determine whether there is a decomposition of the complete graph K_v into copies of F.

We construct solutions whenever F has a sufficiently large odd cycle meeting a specified lower bound and, in addition, F has a single-flip automorphism (i.e. an involutory automorphism acting as a reflection on exactly one cycle). For even orders v, we give analogous results for the maximum packing and minimum covering variants of the problem. We also show a similar result when the edges of K_v have multiplicity 2, but in this case we only require that F has a sufficiently large cycle.

Our methods build on the techniques used in [2] and involve a doubling construction defined in [1] which we apply to graceful labelings of 2-regular graphs with a vertex removed, allowing us to explicitly construct solutions to the Oberwolfach Problem with well-behaved automorphisms.

This is joint work with Andrea Burgess and Peter Danziger.

[1] M. Buratti, T. Traetta. 2-starters, graceful labelings, and a doubling construction for the Oberwolfach Problem. J. Combin. Des. 20 (2012), 483–503.

[2] T. Traetta. A complete solution to the two-table Oberwolfach Problems. J. Combin. Theory Ser. A 120 (2013), 984–997.