Magnetic Helicity and the Calabi Invariant

Gunnar Hornig
University of Dundee
ghornig@dundee.ac.uk
Callum Birkett
University of Dundee
c.birkett@dundee.ac.uk

Magnetic helicity (also called Asymptotic Hopf invariant, V.I Arnold 1974) is an important tool in the study of both astrophysical and laboratory plasmas. Helicity is an integral over the helicity density $h = A \wedge B$, $dA = B$, that measures the average asymptotic linkage of magnetic flux B in a given domain. It is a topological invariant of the magnetic field and provides a lower bound for the energy. However, the use of the helicity integral has been hampered by the fact that it only measures the average linkage over the whole domain and does not provide any more detailed information about linkage within parts of the domain. Attempts to extract more information about the linkage of flux, for instance by considering the helicity density, or line integrals over the helicity density (field line helicity), encounter the problem that h is not gauge invariant. In this talk we introduce the Calabi invariant, an integral quantity closely associated with helicity (Calabi 1970, Gambaudo et al. 2000) and show that this leads to interesting new ways to interpret the helicity integral and allows to calculate a gauge invariant asymptotic field line helicity.

References:

[1] Eugenio Calabi, On the group of automorphisms of a symplectic manifold, from: "Problems in Analysis" (Lectures at the Sympos. in Honor of Salomon Bochner, Princeton Univ., Princeton, NJ, 1969), 1970.

[2] Jean-Marc Gambaudo and Maxime Lagrange, Topological lower bounds on the distance between area preserving diffeomorphisms. Boletim da Socieda de Brasileira de Matematica, 31(1):9–27, 2000.