Densely branching trees as models for Hénon-like and Lozi-like attractors

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Inspired by a recent work of Crovisier and Pujals on mildly dissipative diffeomorphisms of the plane, we show that Hénon-like and Lozi-like maps on their strange attractors are conjugate to natural extensions (a.k.a. shift homeomorphisms on inverse limits) of maps on metric trees with dense set of branch points. In consequence, these trees very well approximate the topology of the attractors, and the maps on them give good models of the dynamics. To the best of our knowledge, these are the first examples of canonical two-parameter families of attractors in the plane for which one is guaranteed such a 1-dimensional locally connected model tying together topology and dynamics of these attractors. For Hénon maps this applies to Benedicks-Carleson positive Lebesgue measure parameter set, and sheds more light onto the result of Barge from 1987, who showed that there exist parameter values for which Hénon maps on their attractors are not natural extensions of any maps on branched 1-manifolds. For Lozi maps the result applies to an open set of parameters given by Misiurewicz in 1980. Our result, that extends to certain attractors that are homoclinic classes for mildly dissipative diffeomorphisms, can be seen as a generalization to the non-uniformly hyperbolic world of a classical result of Williams from 1967.