Refined moves for structure-preserving isomorphisms between graph C^*-algebras

Søren Eilers

University of Copenhagen

`eilers@math.ku.dk`

In work with Restorff, Ruiz and Sørensen, we recently provided a complete description of the equivalence class on the set of graphs with finitely many vertices and at most countably many edges induced by stable isomorphism of their associated graph C^*-algebras. This description was a list of "moves" that generate the equivalence relation in the sense that two such graphs are stably isomorphic if and only if one may transform one into another by a finite number of such moves.

A concurrent program pioneered by Kengo Matsumoto has established strikingly strong rigidity results for Cuntz-Krieger algebras when one considers them not just as C^*-algebras, but also take finer structure (a certain Abelian subalgebra and a certain circle action) into account, showing in a multitude of ways that operator algebraic objects completely remember the dynamics underlying their definition.

Since the Cuntz-Krieger algebras lie in the class of graph C^*-algebras, it is a natural (one might even say: pressing) question if a similar combinatorial description may be obtained for the finer equivalence relations induced by structure-preserving isomorphism of the graph C^*-algebras, and we present a list of moves which we conjecture solves this problem in the very satisfactory sense that the subset of moves preserving extra structure in fact also generates the relevant equivalence relation.

We can prove the conjecture for all graphs defining simple Cuntz-Krieger algebras, and in a large number of special cases beyond that. Apart from the core results obtained with Ruiz, I will also present recent work obtained with Becky Armstrong, Kevin Brix, Toke Meier Carlsen, Aidan Sims, and Gábor Szabó.