# Numerical computation of the complex zeros of Bessel and Hankel functions

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The complex zeros of cylinder functions appear in several problems of applied mathematics and theoretical physics. For example, the complex zeros of Hankel functions are involved in quantum scattering problems by spheres and cylinders. An algorithm (with a Matlab implementation) for computing the complex zeros of the Bessel function of first kind $J_\nu(z)$, second kind $Y_\nu(z)$, Hankel functions $H_\nu^{(1)}(z)$, $H_\nu^{(2)}(z)$ and general combinations of Bessel functions $\alpha J_\nu(z) + \beta Y_\nu(z)$ and Hankel functions $\alpha H_\nu^{(1)}(z) + \beta H_\nu^{(2)}(z)$ is described in this presentation. The algorithm, based on the results obtained in [1], [2], allows to obtain with certainty and accuracy all the zeros of the selected function inside a box in the complex plane. The performance of the algorithm is illustrated with numerical examples. This is a joint work in collaboration with Amparo Gil and Javier Segura.

**References**

[1] Computing the complex zeros of special functions. J. Segura, Numerische Mathematik 124(4), 2013, pp 723-752

[2] On the complex zeros of Airy and Bessel functions and those of their derivatives. A. Gil, J. Segura. Anal. Appl. 12(5) (2014) 573-561

[3] An algorithm for computing the complex zeros of Bessel and Hankel functions A. Gil, D. Ruiz-Antolin, J. Segura In progress