Local asymptotics and unique continuation from boundary points for fractional equations

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In this talk I will present some results in collaboration with A. De Luca and S. Vita on unique continuation and local asymptotics of solutions to fractional elliptic equations at boundary points, under some outer homogeneous Dirichlet boundary conditions. I will describe a blow-up procedure which involves an Almgren type monotonicity formula and provides a classification of all possible homogeneity degrees of limiting entire profiles. The Caffarelli-Silvestre extension provides an equivalent formulation of the fractional equation as a local degenerate or singular problem in one dimension more, with mixed Dirichlet and Neumann boundary conditions. In the development of a monotonicity argument, the mixed boundary condition raises delicate regularity issues, which turn out to be quite difficult in dimension $N \geq 2$ due to the positive dimension of the junction set and some role played by the geometry of the domain. Such difficulties are overcome by a double approximation procedure: by approximating the potential with functions vanishing near the boundary and the Dirichlet N-dimensional region with smooth $(N + 1)$-dimensional sets with straight vertical boundary, it is possible to construct a sequence of approximating solutions which enjoy enough regularity to derive Pohozaev type identities, needed to obtain Almgren type monotonicity formulas and consequently to perform blow-up analysis.