Smooth compactifications of differential graded categories

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We will give an overview of results on smooth categorical compactifications, the questions of their existence and their construction. The notion of a categorical smooth compactification is a straightforward generalization of the corresponding usual notion for algebraic varieties.

First, we will explain the result on the existence of smooth compactifications of derived categories of coherent sheaves on separated schemes of finite type over a field of characteristic zero. Namely, such a derived category can be represented as a quotient of the derived category of a smooth projective variety, by a triangulated subcategory generated by a single object. Then we will give an example of a homotopically finite DG category which does not have a smooth compactification: a counterexample to one of the Kontsevich's conjectures on the generalized Hodge to de Rham degeneration.

If time permits, we will formulate a K-theoretic criterion for existence of a smooth categorical compactification, using a DG categorical analogue of Wall's finiteness obstruction from topology.