Weighted composition operators via hyperbolic C_0-groups on $\mathbb{D}$

Jesús Oliva Maza
Universidad de Zaragoza
joliva@unizar.es
Luciano Abadías
Universidad de Zaragoza
labadias@unizar.es
José E. Galé
Universidad de Zaragoza
gale@unizar.es
Pedro J. Miana
Universidad de Zaragoza
pjmiana@unizar.es

In this talk, we present our recent work regarding a family of C_0-groups of weighted composition operators on Hardy and Bergman spaces on the disk, i.e. $f \mapsto \frac{k\circ\varphi_t}{k} f \circ \varphi_t$, where $(\varphi_t)_{t\in\mathbb{R}}$ is a group of hyperbolic automorphisms of the disk $\mathbb{D}$ onto itself, and $k : \mathbb{D} \to \mathbb{C}$ is a holomorphic function with polynomial limits of arbitrary order $\alpha, \beta \in \mathbb{R}$ at the fixed points of $(\varphi_t)_{t\in\mathbb{R}}$.

In the first part, we are able to characterize in detail the spectra of the infinitesimal generator of the C_0-group, which only depends on the values of α and β. More precisely, we obtain its point spectra, approximate point spectra and essential spectra. This is done by using a mixture of tools of C_0-semigroups theory, and the theory of weighted composition operators on these spaces of holomorphic functions.

After that, an application of the spectral mapping theorem is shown with some subordinated operators, some of which resemble the Hilbert or Cesàro operators. In a final remark, we show that one cannot take these C_0-groups to $\ell^p(\mathbb{N}_0)$ spaces since they do not define bounded operators there for $p \neq 2$.