

Constructive Method to Solving 3D Navier - Stokes Equations

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Abstract

We consider 3D Navier – Stokes equations for motion of incompressible media. These equations are of mathematical interest and have a lot of applications to practical problems. For today many aspects connected with the Navier – Stokes equations have been studied not enough and need more profound investigation [1-2]. Main unresolved problem is the lack of a constructive method of solution. How to resolve the Navier – Stokes equations while preserving all nonlinear terms is the question that needs to be addressed.

The author proposes an approach to this problem the essence of which is to reduce the basic problem of solving the Navier – Stokes equations to a set of simple tasks. We face to five more simple tasks that you need to consistently allow. Each of the individual Navier – Stokes equations must be reduced to a free divergent form and integrated. Resulting equality can be converted so as to exclude some nonlinear and non-divergent terms. As the result we arrive to nine equations which link main unknowns u, v, w, p , associated unknowns Ψ_i where $i = 1, 2, \dots, 15$ and an arbitrary additive functions in three variables $\alpha_i, \beta_i, \gamma_i, \delta_i$. Considered together these nine ratios provide the first integral of 3D Navier – Stokes equations [3]. The well-known integrals of Bernoulli, Euler - Bernoulli and Lagrange - Cauchy are its special cases [4]. Four of the nine obtained equations represent expressions for basic unknowns. So they determine the overall structure of the solutions. The remaining five equations can be resolved relative to six associated unknowns Ψ_j where $j = 10, 11, \dots, 15$ only if two conditions of compatibility are hold. They reduce to two fifth order equations with respect to nine unknown Ψ_k where $k = 1, 2, \dots, 9$. Each set of functions Ψ_k satisfying a given system determines exact solution of the Navier - Stokes equations. To complete the solution you need to find the six remaining associated unknowns Ψ_j , where $k = 10, 11, \dots, 15$. Three of the last functions can be set arbitrarily whereas the remaining three are defined as solution of linear inhomogeneous equations.

As a result all values presents of the structure formula for main unknowns are defined and these unknowns are easy to find. Some exact

solutions constructed in this way are given in [5-6]. A similar approach can be applied to construct solutions of the Euler equations for case of motion for inviscid incompressible media. In all relationships it is enough to put $\frac{1}{Re} = 0$.

References

- [1] Lodizhenskaya O.A., The Mathematical Theory of Viscous Incompressible Fluid , *Gordon and Breach, New York, USA*, 1969.
- [2] Charls L. Fefferman, Existence and Smoothness of the Navier — Stokes Equation, *Preprint, Princeton Univ., Math. Dept., Princeton, New Jzersy, USA*, 2000, pp. 1-5.
- [3] Koptev A.V., Integrals of Motion of an Incompressible medium Flow. From Classic to Modern. In book: Handbook on Navier — Stokes Equations. Theory and Applied Analysis , Nova Science Rublishers, New York, USA 2017, pp.443-460.
- [4] Koptev A.V., Systematization and analysis of integrals of motion for an incompressible fluid flow, *Jour. of Siberian Federal Univ., Math. and Phys.*, **2018**, 11(3), P. 370-382.
- [5] Koptev A.V., Exact solutions of 3D Navier — Stokes equations, *Jour. of Siberian Federal Univ., Math. and Phys.*, **2020**, 13(3), P. 306-313.
- [6] Koptev A.V., Method for Solving the Navier — Stokes and Euler Equations of Motion for Incompressible Media, *Jour. of Mathematical Sciences, Springer, New York, USA.*, **2020**, Vol. 250(1), P. 254-382.