Quantum polynomial optimisation problems for dimension d variables, with symmetries

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Quantum Information Theory (QIT) involves quantum states and measurements, mathematically represented as non-commutative positive operators. A typical problem in QIT is to find the minimum of a polynomial expression in these operators. For instance, finding the maximum violation of a Bell inequality, or the ground state energy of a Hamiltonian are polynomial optimisation problems in non-commuting variables. The NPA hierarchy [New J. Phys. 10, 073013 (2008)], which can be viewed as the "eigenvalue" version of Lasserre's hierarchy, provides a converging hierarchy of SDP relaxation of a non-commutative polynomial optimisation problem involving variables of unbounded dimension. This hierarchy converges, it is one of QIT main technical tool.

Importantly, some QIT problems concern operators of bounded dimension d. The NPA hierarchy was extended into the NV hierarchy [Phys. Rev. Lett.115, 020501(2015)] to tackle this case. In this method, one first sample at random many dimension d operators satisfying the constraint, and compute the associated moment matrix. This first step discovers the moment matrix vector space, over which the relaxed SDP problem is solved in a second step. In this talk, we will first review this method. Then, based on [Phys. Rev. Lett. 122, 070501], we will show how one can reduce the computational requirements by several orders of magnitude, exploiting the eventual symmetries present in the optimization problem.