Dual nonnegativity certificates and efficient algorithms for rational sum-of-squares decompositions

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We study the problem of computing rational weighted sum-of-squares (WSOS) certificates for positive polynomials over a compact semialgebraic set. In the first part of the talk, we introduce the concept of dual cone certificates, which allows us to interpret vectors from the dual of the WSOS cone as rigorous nonnegativity certificates. Every polynomial in the interior of the WSOS cone admits a full-dimensional cone of dual certificates; as a result, rational WSOS certificates can be constructed from numerically computed dual certificates at little additional cost. In the second part of the talk, we use this theory to develop an almost entirely numerical hybrid algorithm for computing the optimal WSOS lower bound of a given polynomial along with a rational dual certificate, with a polynomial-time computational cost per iteration and linear rate of convergence. As a special case, we obtain a new polynomial-time algorithm for certifying the nonnegativity of strictly positive polynomials over an interval.