

Strict positive definiteness of non-radial kernels on d -dimensional spheres

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Isotropic positive definite functions are used in approximation theory and are for example applied in geostatistics and physiology. They are also of importance in statistics where they occur as correlation functions of homogeneous random fields on spheres. We study a class of function applicable for interpolation of arbitrary scattered data on \mathbb{S}^{d-1} by linear combinations of a kernel $K : \mathbb{S}^{d-1} \times \mathbb{S}^{d-1} \rightarrow \mathbb{C}$ evaluated at the interpolation points in the second argument. The isotropic kernels are a special case of this approach and we study kernels with more general properties like axial symmetry and invariance under parity.

A class of kernels for which the resulting interpolation problem is uniquely solvable for any distinct point set $\Xi \subset \mathbb{S}^d$ are known strict positive definite isotropic functions. Using recent results of Bonfim and Menegatto [2] and the famous representations of isotropic positive definite functions on \mathbb{S}^{d-1} due to Schoenberg as starting point we derive new sufficient conditions for strict positive definiteness of axial symmetric and convolutional kernels. The results extend a necessary and sufficient characterisation of strict positive definite isotropic basis functions by Chen et al. proven in [1] to a non-radial kernel class.

References

- [1] Chen, D. and Menegatto, V. A. and Sun, X.: *A necessary and sufficient condition for strictly positive definite functions on spheres*, Proceedings of the American Mathematical Society, 131, 2733-2740, 2003.
- [2] Bonfim, R. and Menegatto, V., *Strict positive definiteness of multivariate covariance functions on compact two-point homogeneous spaces*, Journal of Multivariate Analysis, 152, 2016.