Optimal Grid Drawings of Complete Multipartite Graphs and an Integer Variant of the Algebraic Connectivity

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We use spectral graph theory to show how to draw the vertices of a complete multipartite graph G on different points of a bounded d-dimensional integer grid, such that the sum of squared distances between vertices of G is (i) minimized or (ii) maximized. For both problems we provide a characterization of the solutions. For the particular case d = 1, our solution for (i) also settles the minimum-2-sum problem for complete bipartite graphs; the minimum-2-sum problem was defined by Juvan and Mohar in 1992. Weighted centroidal Voronoi tessellations are the solution for (ii). Such drawings are related with Laplacian eigenvalues of graphs. This motivates us to study which properties of the algebraic connectivity of graphs carry over to the restricted setting of drawings of graphs with integer coordinates.