Wigner's theorem in normed spaces

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Let $(H, (\cdot, \cdot))$ and $(K, (\cdot, \cdot))$ be inner product spaces over $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}\}$ and suppose that $f\colon H \to K$ is a mapping satisfying

$$|(f(x), f(y))| = |(x, y)|, \quad x, y \in H. \tag{1}$$

Then the famous Wigner's unitary–antiunitary theorem says that f is a solution of (1) if and only if it is phase equivalent to a linear or an anti-linear isometry, say U, that is,

$$f(x) = \sigma(x)Ux, \quad x \in H,$$

where $\sigma\colon H \to \mathbb{F}$, $|\sigma(x)| = 1$, $x \in H$, is a so called phase function. In this talk several generalizations of this theorem to the setting of normed spaces will be presented.