The Number of Solutions to $ax + by + cz = n$ and its Relation to Quadratic Residues

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We find a very efficient formula for calculating the number of solutions of the equation $ax + by + cz = n$ in non-negative integer triples (x, y, z), where a, b, c and n are given natural numbers. This formula involves some summations of floor functions of fractions. To quickly evaluate these sums, we find a reciprocity relation which generalizes a well-known reciprocity relation of Gauss, related to the law of quadratic reciprocity. Further, by counting the number of solutions of the equation $px + qy + z = \frac{q(p-1)}{2}$ in two different ways, we prove that the above result of Gauss is equivalent to a well-known result of Sylvester related to the Frobenius Coin Problem. This work has been published in the Journal of Integer Sequences.