Fractional Integrals with Measure in Grand Lebesgue and Morrey spaces

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In the last two decades, the theory of grand Lebesgue spaces $L^{p)}$ introduced by T. Iwaniec and C. Sbordone [IS] is one of the intensively developing directions in modern analysis. The necessity to investigate these spaces emerged from their rather essential role in various fields, in particular, in the integrability problem of Jacobian under minimal hypotheses. It turns out that in the theory of PDEs, the generalized grand Lebesgue spaces $L^{p),\theta}$ introduced by Greco, Iwaniec, and Sbordone [GIS] are appropriate for treating the existence and uniqueness, as well as the regularity problems for various non-linear differential equations.

The aim of our talk is to give a complete characterization of a class of measures μ governing the boundedness of fractional integral operators I^{γ} defined on a quasi-metric measure space (X, d, μ) (nonhomogeneous space) from one grand Lebesgue spaces $L_{\mu}^{p),\theta_1}(X)$ into another one $L_{\mu}^{q),\theta_2}(X)$. As a corollary, we have a generalization of the Sobolev inequality for potentials with measure. D. Adams trace inequality (i.e., $L_{\mu}^{p),\theta_1}(X) \mapsto L_{\nu}^{q),\theta_2}(X)$ boundedness) is also derived for these operators in grand Lebesgue spaces. Appropriate problems for grand Morrey spaces are also studied. In the case of Morrey spaces, we assume that the underlying sets of spaces might be of infinite measure. Under some additional conditions on a measure, we investigate the sharpness of the second parameter θ_2 in the target space.

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References

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