G_2- and $Spin(7)$-structures by means of vector cross products

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The notion of a multilinear vector cross product (VCP) has been introduced by Gray as a natural generalization of the notion of an almost complex structure. Gray also associated G_2-structures on 7-manifolds and $Spin(7)$-structures on 8-manifolds with VCPs on the underlying manifolds. In my talk I shall present recent results on G_2-structures and $Spin(7)$-structures using VCPs: 1) the correspondence between parallel VCPs on a Riemannian manifold M and parallel almost complex structures on a higher dimensional knot space over M endowed with a L^2-metric, which generalizes Brylinski's, LeBrun's, Henrich's and Verbitsky's results for the case that S is a codimension 2 submanifold in M, and $S = S^1$ or M is a torsion-free G_2-manifold, respectively; 2) the similarities between integrable complex structures on one hand and torsion-free G_2-and Spin(7)-structures on the other hand; 3) the construction of CR-twistor spaces over G_2-manifolds, due to Verbitsky, and its extension to $Spin(7)$-manifolds. My lecture is based on my joint works with Domenico Fiorenza, Kotaro Kawai, Lorenz Schwachhöfer and Luca Vitagliano.