A proof of a conjecture of Elbert and Laforgia on the zeros of cylinder functions

Gergő Nemes

Alfréd Rényi Institute of Mathematics

nemes.gergo@renyi.hu

We prove the enveloping property of the known divergent asymptotic expansion of the large real zeros of the cylinder functions, and thereby answering in the affirmative a conjecture posed by Elbert and Laforgia in 2001 (*J. Comput. Appl. Math.* **133** (2001), no. 1–2, p. 683). The essence of the proof is the construction of an analytic function that returns the zeros when evaluated along certain discrete sets of real numbers. By manipulating contour integrals of this function, we derive the asymptotic expansion of the large zeros truncated after a finite number of terms plus a remainder that can be estimated efficiently. The conjecture is then deduced as a corollary of this estimate.