Positive maps and trace polynomials from the symmetric group

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With techniques borrowed from quantum information theory, we develop a method to systematically obtain operator inequalities and identities in several matrix variables. These take the form of trace polynomials: polynomial-like expressions that involve matrix monomials $X_{\alpha_1} \cdots X_{\alpha_r}$ and their traces $\mathrm{tr}(X_{\alpha_1} \cdots X_{\alpha_r})$. Our method rests on translating the action of the symmetric group on tensor product spaces into that of matrix multiplication. As a result, we extend the polarized Cayley-Hamilton identity to an operator inequality on the positive cone, characterize the set of multilinear equivariant positive maps in terms of Werner state witnesses, and construct permutation polynomials and tensor polynomial identities on tensor product spaces. We give connections to concepts in quantum information theory and invariant theory.