Shape optimization of light structures and the vanishing mass conjecture

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It is a classical problem in the theory of shape optimization to find a shape with minimal (linear) elastic compliance (or, equivalently, maximal stiffness) for a given amount of mass and prescribed external forces. It is an intriguing question with a long history, going back to Michell's seminal 1904 work on trusses, to determine what happens in the limit of vanishing mass. Contrary to all previous results, which utilize a soft mass constraint by introducing a Lagrange multiplier, we here consider the hard mass constraint. Our results establish the convergence of approximately optimal shapes of (exact) size tending to zero to a limit generalized shape represented by a (possibly diffuse) probability measure. This limit generalized shape is a minimizer of the limit compliance, which involves a new integrand, namely the one conjectured by Bouchitte in 2001 and predicted heuristically before in works of Allaire-Kohn (80's) and Kohn-Strang (90's). This integrand gives the energy of the limit generalized shape understood as a fine oscillation of (optimal) lower-dimensional structures. Its appearance is surprising since the integrand in the original compliance is just a quadratic form and the non-convexity of the problem is not immediately obvious. I will also present connections to the theory of Michell trusses and show how our results can be interpreted as a rigorous justification of that theory on the level of functionals in both two and three dimensions. This is joint work with J.F. Babadjian (Paris-Saclay) and F. Iurlano (Paris-Sorbonne).