The topological conjugacy criterion for surface Morse-Smale flows with a finite number of moduli

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The Morse-Smale flows have been classified in sense of topological equivalence for several times during the last century. The most known invariants for such flows are invariants by E. Leontovich and A. Mayer [1], [2], M. Peixoto [3], A. Oshemkov and V. Sharko [4]. Besides, the Ω-stable flows on surfaces have been classified in such sense too by D. Neumann and T. O'Brien [5] and V. Kruglov, D.Malyshev and O. Pochinka [6]. Attempts were also made to classify Morse-Smale flows in sense of topological conjugacy: in particular, V. Kruglov [7] proved that the classes of topological equivalence and topological conjugacy for gradient-like flows on surfaces coincide.

J. Palis [8] considered a flow in a neighbourhood of a separatrix which connects two saddle points. He showed that in each topological equivalence class there is continuum of topological conjugacy classes, that is a flow with a separatrix-connection has analitical conjugacy invariants called *moduli of stability* or *moduli of topological conjugacy.* Each limit cycle likewise generates at least one modulus associated with its period. V. Kruglov, O. Pochinka and G. Talanova [9] proved that non-singular flows on an annulus with only two limit cycles on the annulus's boundary components have infinite number of moduli, expressed by a function.

The first result of this report is the following.

THEOREM. *A Morse-Smale surface flow has finite number of moduli iff it has no a trajectory going from one limit cycle to another.*

To construct the topological classification in sense of conjugacy we use the complete topological classifications in sense of equivalence from [4], [6]. Namely, there is one-to-one correspondence between equivalent classes of a Morse-Smale flow ϕ^t on a surface and isomorphic classes of the *equipped graph* $\Upsilon^*_{\phi^t}$, which contains an information about partition the ambient manifold into cells with similar trajectories behaviour and the limit cycles types.

To distinguish topological conjugacy classes we add to the equipped graph an information on the periods of the limit cycles. It gives a new equipped graph $\Upsilon^{**}_{\phi^t}$, and here is the second result.

THEOREM. *Morse-Smale surface flows ϕ^t, ϕ'^t without trajectories going from one limit cycle to another one are topologically conjugate iff their equipped graphs $\Upsilon^{**}_{\phi^t}$ and $\Upsilon^{**}_{\phi'^t}$ are isomorphic.*

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